

We'll start with the simplest substitution problem, which turns out to be very useful in calculus.

Solution: Since the two expressions are both equal to y , they must be equal to each other:

$$\Rightarrow \boxed{x = \frac{5}{3}}$$

By the way, we've just calculated the x-coordinate of the *point of intersection* of the lines $y = 3x - 7$ and $y = -9x + 8$.

❑ FRACTIONS NOT NEEDED

EXAMPLE 2: Solve the system:

$$\begin{aligned} 3x + y &= 12 \\ -5x + 2y &= 13 \end{aligned}$$

Solution: The command “Solve the system” means we must find the values of both x and y that make both equations true.

The first step is to choose an equation and a variable within that equation to solve for. For this system, the best variable to solve for is the y in the first equation. Why y ? Because its coefficient is 1, which means we avoid fractions when solving for y .

Solving the first equation for y gives us

$$y = 12 - 3x \quad (\text{by subtracting } 3x \text{ from each side})$$

Now we substitute the expression $12 - 3x$ into the second equation where the y is.

$$\begin{aligned} -5x + 2y &= 13 && (\text{the second equation}) \\ \Rightarrow -5x + 2(\mathbf{12 - 3x}) &= 13 && (\text{since } y = 12 - 3x) \\ \Rightarrow -5x + 24 - 6x &= 13 && (\text{distribute}) \\ \Rightarrow -11x + 24 &= 13 && (\text{combine the } x\text{'s}) \\ \Rightarrow -11x &= -11 && (\text{subtract } 24 \text{ from each side}) \\ \Rightarrow \underline{x = 1} &&& (\text{divide each side by } -11) \end{aligned}$$

Since $y = 12 - 3x$, and we just found out that $x = 1$, we can solve for y :

$$\begin{aligned} y &= 12 - 3(\mathbf{1}) \\ y &= 12 - 3 \\ \underline{y} &= \mathbf{9} \end{aligned}$$

$x = 1 \text{ \& } y = 9$

Notes: First, if you’d like to check our solution, substitute the values of x and y into both of the original equations. Second, what if none of the variables has a coefficient of 1? Well, you’ll just have to

pick a variable and then deal with whatever fractions may arise. In the real world, however, it's probably best to solve the system using the Elimination Method.

Homework

1. Solve each system using the Substitution Method, and be sure you practice checking your solution (your pair of numbers) in both of the original equations:

a.
$$\begin{aligned} 2x + y &= 5 \\ -2x + 7y &= 19 \end{aligned}$$

b.
$$\begin{aligned} 3m - 2n &= 34 \\ -6m + n &= -62 \end{aligned}$$

c.
$$\begin{aligned} -3w + 4m &= 6 \\ -3w - m &= 1 \end{aligned}$$

❑ **FRACTIONS NEEDED**

EXAMPLE 2: Solve the system:
$$\begin{aligned} 5x + 3y &= 7 \\ 4x - 2y &= 1 \end{aligned}$$

Solution: There's no way to avoid fractions (can you see why?), so even though we could solve for either variable in either equation, for no particular reason we'll solve for x in the first equation:

$$5x + 3y = 7 \quad (\text{the first equation})$$

$$\Rightarrow 5x = -3y + 7 \quad (\text{subtract } 3y \text{ from each side})$$

$$\Rightarrow x = \frac{-3y+7}{5} \quad (*) \quad (\text{divided each side by } 5)$$

Now substitute this value of x into the second equation:

$$4\left(\frac{-3y+7}{5}\right) - 2y = 1$$

$$\Rightarrow \frac{-12y+38}{5} - 2y = 1$$

Multiply both sides of the equation (that is, all three terms) by 5:

$$\Rightarrow \left(\frac{-12y+28}{5} \right) [5] - 2y [5] = 1 [5]$$

$$\Rightarrow \left(\frac{-12y+28}{\cancel{5}} \right) [\cancel{5}] - 2y [5] = 1 [5]$$

$$\Rightarrow -12y + 28 - 10y = 5$$

$$\Rightarrow -22y = -23$$

$$\Rightarrow y = \frac{23}{22}$$

Sorry, but we're not done — we still need to find x , so we substitute the value of y just obtained into the equation for x (the one with the * next to it):

$$x = \frac{-3y+7}{5}$$

And since $y = \frac{17}{22}$, it follows that (hold on tight!)

$$x = \frac{-3\left(\frac{23}{22}\right)+7}{5} = \frac{\frac{-69}{22}+7}{5} = \frac{\frac{-69}{22}+\frac{7(22)}{22}}{5} = \frac{\frac{-69}{22}+\frac{154}{22}}{5} = \frac{\frac{85}{22}}{5}$$

$$= \frac{85}{22} \times \frac{1}{5} = \frac{5 \cdot 17}{22} \times \frac{1}{5} = \frac{\cancel{5} \cdot 17}{22} \times \frac{1}{\cancel{5}} = \frac{17}{22}$$

And so our final solution is

$x = \frac{17}{22} \text{ and } y = \frac{23}{22}$
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Homework

2. Solve each system using the Substitution Method:

$$\begin{array}{lll} \text{a.} & \begin{array}{l} 2x + 3y = 7 \\ 4x - 2y = 11 \end{array} & \text{b.} \begin{array}{l} 5a - 2b = 5 \\ 4a - 3b = 9 \end{array} & \text{c.} \begin{array}{l} 2w + 3z = 7 \\ w - 3z = 13 \end{array} \end{array}$$

Solutions

$$\begin{array}{lll} \mathbf{1.} & \text{a.} & x = 1, y = 3 & \text{b.} & m = 10, n = -2 & \text{c.} & w = -\frac{2}{3}, m = 1 \\ \mathbf{2.} & \text{a.} & x = \frac{47}{16}, y = \frac{3}{8} & \text{b.} & a = -\frac{3}{7}, b = \frac{-25}{7} & \text{c.} & w = \frac{20}{3}, z = \frac{-19}{9} \end{array}$$

“There are no
shortcuts to
any place
worth going.”

– Beverly Sills, Opera Soprano